The use of optimal control theory for equilibrium identification and optimization of plasma scenarios

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Journée Méthodes Numériques pour les Plasmas, Marseille

Outline

1. Inverse problems
   1. plasma boundary identification
   2. full equilibrium reconstruction
2. Optimization of plasma scenarios
3. Perspective : coupling with transport

Equilibrium identification. Introduction

- Equilibrium of a plasma : a free boundary problem
- Equilibrium equation inside the plasma, in an axisymmetric configuration : Grad-Shafranov equation
- Right-hand side of this equation is a non-linear source : the toroidal component of the plasma current density

Goal

Perform the reconstruction of 2D equilibrium and the identification of the current density in real-time.

Mathematical modelling of the equilibrium

Grad-Shafranov Equation

- 3D MHD equilibrium + axisymmetric assump. : Grad-Shafranov eqn.
- 2D problem. State variable $\psi(r, z)$ poloidal magnetic flux

In the plasma

$$-\Delta^* \psi = rp'(\psi) + \frac{1}{\mu_0 r}(ff')(\psi)$$

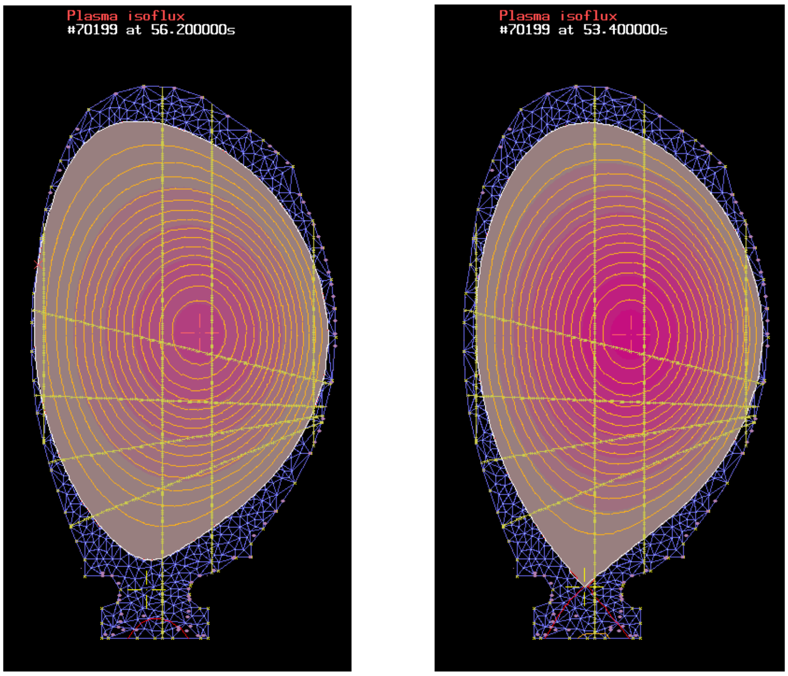
with

$$\Delta^*. = \frac{\partial}{\partial r}(\frac{1}{\mu_0 r}\frac{\partial .}{\partial r}) + \frac{\partial}{\partial z}(\frac{1}{\mu_0 r}\frac{\partial .}{\partial z})$$

In the vacuum

$$-\Delta^* \psi = 0$$

Definition of the free plasma boundary

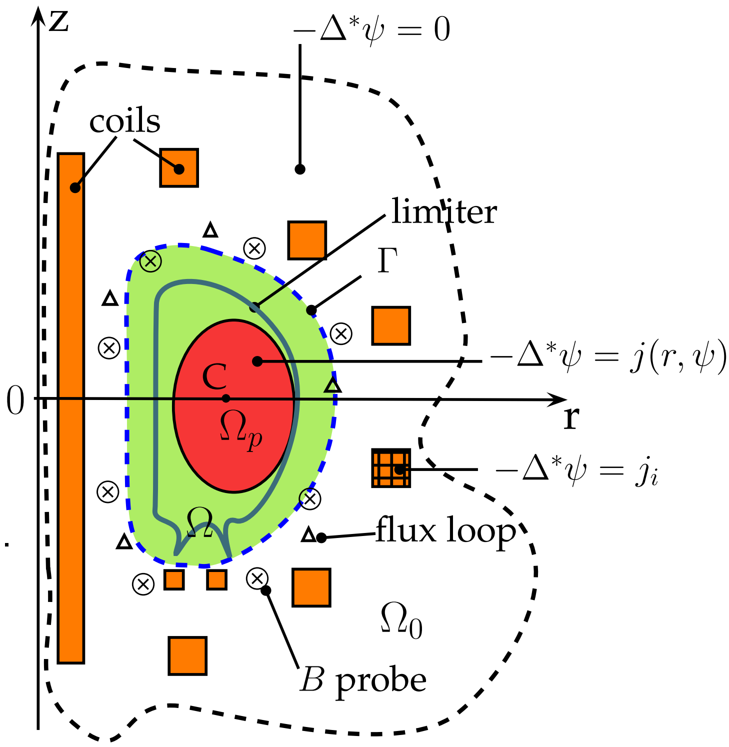


Two cases

- outermost flux line inside the limiter (left)
- magnetic separatrix : hyperbolic line with an X-point (right)

VacTH

- Solves the external problem. ψ outside the plasma.
- Decomposition of ψ in toroidal harmonics
- Interpolation of discrete magnetic measurements on a fixed contour Γ
  - Boundary conditions for equilibrium reconstruction
  - Generic. Use on different Tokamaks. IMAS.
- Extrapolation for plasma boundary reconstruction (WEST)



Decomposition of ψ in any annular domain surrounding the plasma

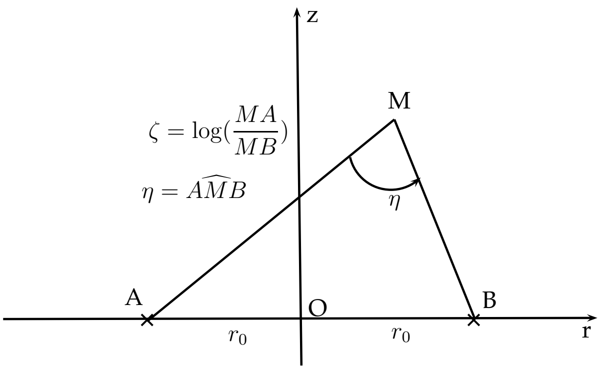
$$\psi = \psi_C + \psi_{TH}$$

- ψ_C contribution of PFcoils. Green functions.
- ψ_{TH} satisfies

$$\Delta^* \psi_{TH} = 0$$

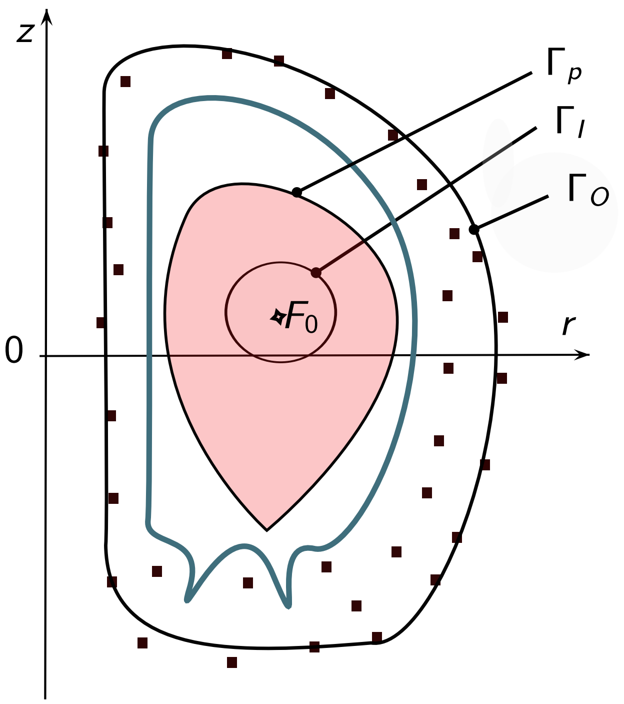
and can be uniquely decomposed in a series of toroidal harmonics

$$\begin{cases} \psi_{TH} = \psi_{ext} + \psi_{int} \\ \psi_{ext} = \dfrac{r_0 \sinh \zeta}{\sqrt{\cosh \zeta - \cos \eta}} [\sum_{n=0}^{\infty} (a_n^e \cos(n\eta) + b_n^e \sin(n\eta)) Q_{n-1/2}^1(\cosh \zeta)] \\ \psi_{int} = \dfrac{r_0 \sinh \zeta}{\sqrt{\cosh \zeta - \cos \eta}} [\sum_{n=0}^{\infty} (a_n^i \cos(n\eta) + b_n^i \sin(n\eta)) P_{n-1/2}^1(\cosh \zeta)] \end{cases}$$



toroidal coordinates (ζ, η)

Plasma boundary identification by an optimal control method



- External contour Γ_O with Cauchy conditions (f, g) from Toroidal Harmonics
- Internal contour Γ_I around current center
- Solve equation for ψ in fixed annular domain Ω
- 2 sub-problems Dir-Dir or Neu-Dir :

$$\begin{cases} \Delta^* \psi_D = 0 & \text{in } \Omega \\ \psi_D = f & \text{on } \Gamma_O \\ \psi_D = v & \text{on } \Gamma_I \end{cases} \qquad \begin{cases} \Delta^* \psi_N = 0 & \text{in } \Omega \\ \dfrac{1}{r}\dfrac{\partial \psi_N}{\partial n} = g & \text{on } \Gamma_O \\ \psi_N = v & \text{on } \Gamma_I \end{cases}$$

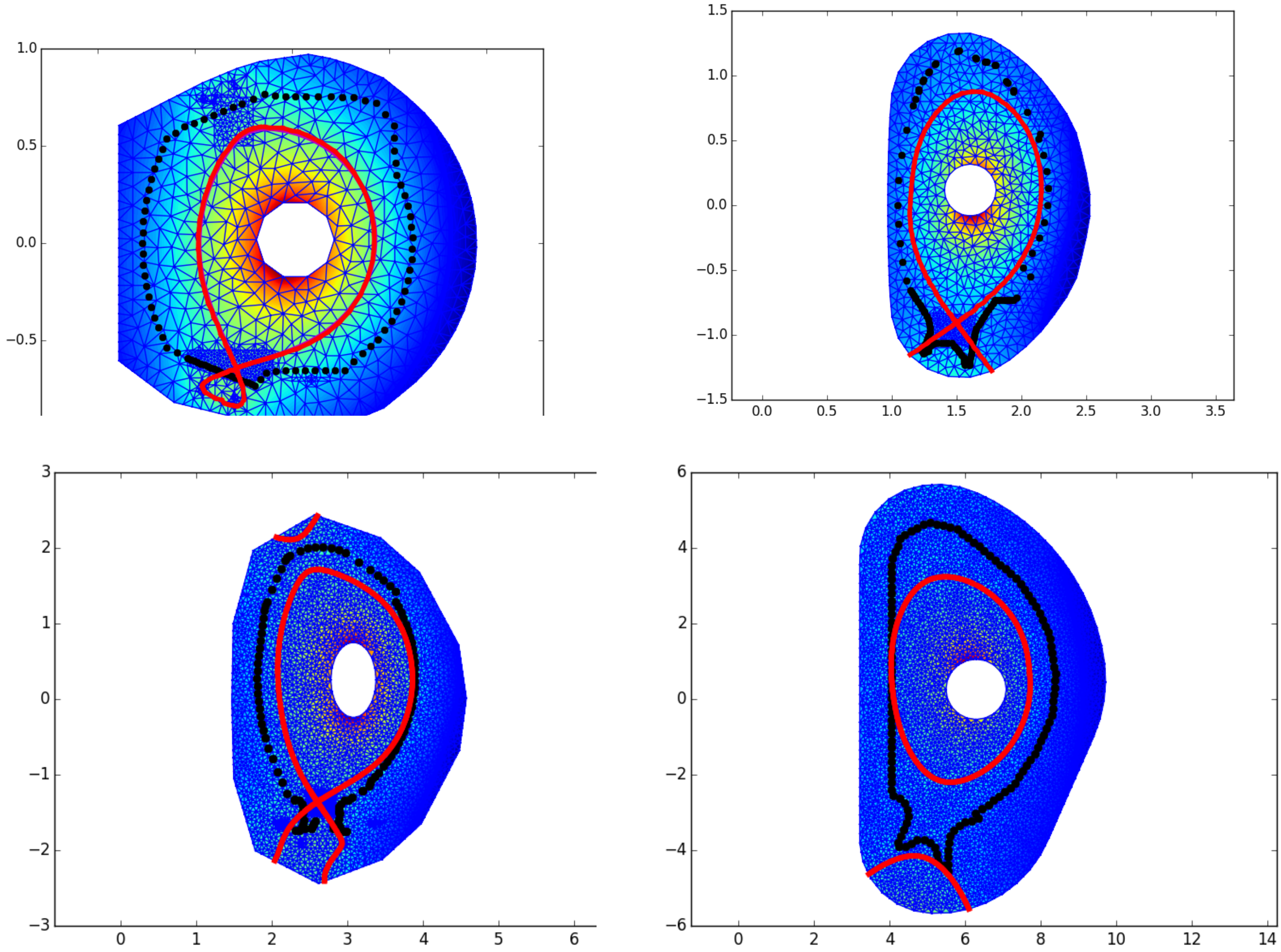
- Minimize a regularized Kohn-Vogelius cost function

$$J(v) = \frac{1}{2}\int_\Omega \frac{1}{r}||\nabla\psi_D(v,f) - \nabla\psi_N(v,g)||^2 dx + \frac{\varepsilon}{2}\int_\Omega \frac{1}{r}||\nabla\psi_D(v,f)||^2 dx$$

$$J(v) = \frac{1}{2}((1+\varepsilon)s_D(v,v) - s_N(v,v)) - l(v) + c$$

Euler equation : $(J'(u), v) = (1+\varepsilon)s_D(u,v) - s_N(u,v) - l(v) = 0 \quad \forall v$

WEST, AUG, JET, ITER, . . .



VacTH algorithm and code

Initialization

- Geom. data : current filament position (ψ_C), flux loops, B probes
- Choice of fixed contour Γ
- Number of harmonics n^e, n^i, ...

One equilibrium. Input data : PF coils currents I_{C_i}, magnetics ψ_i, B_i

- Compute flux $\psi_{C,i}$ and field $B_{C,i}$ generated by the PF coils and substract it from measurements
- Find optimal toroidal harmonics expansion coefficients
  $(a^e_{0:n_e}, b^e_{1:n_e}, a^i_{0:n_i}, b^i_{1:n_i}) = u_{opt} = \mathrm{argmin}\ J(u)$
  with $J(u) = J_{obs}(u) + \varepsilon R(u)$
  - J_{obs} distance to measurements
  - regularization $R(u) = \int_{C_{\zeta_0}} |\frac{d^2\psi_{th}}{ds^2}|^2 ds,$
    C_{ζ_0} circle of constant ζ coordinate surrounding the pole of the coordinates system.
- $\psi = \psi_C + \psi_{TH}(u_{opt})$.
  - interpolation : evaluate Cauchy conditions on Γ
  - but also extrapolation : X-point, plasma boundary, ...

Full equilibrium reconstruction : experimental measurements

- magnetic "measurements" on mesh boundary

$$\psi(M_i) = g_i \text{ and } \frac{1}{r}\frac{\partial \psi}{\partial n}(N_j) = h_j \text{ on } \partial\Omega$$

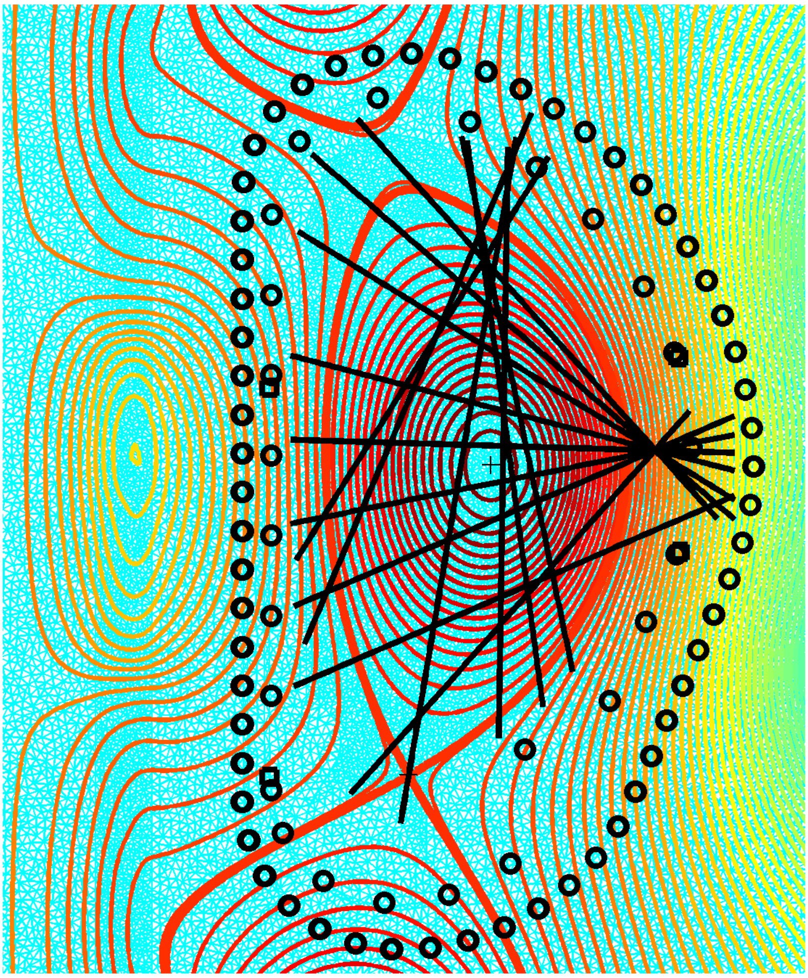
- interferometry and polarimetry on several chords

$$\int_{C_m} n_e(\psi)dl = \alpha_m, \int_{C_m} \frac{n_e(\psi)}{r}\frac{\partial \psi}{\partial n}dl = \beta_m$$

- motional Stark effect

$$f_j(B_r(M_j), B_z(M_j), B_\phi(M_j)) = \gamma_j$$

ITER, magnetic sensors and interferometry-polarimetry chords



Statement of the inverse problem

- State equation

$$\begin{cases} -\Delta^*\psi = \lambda[\frac{r}{R_0}A(\bar{\psi}) + \frac{R_0}{r}B(\bar{\psi})]1_{\Omega_p(\psi)} & \text{in } \Omega \\ \psi = g & \text{on } \partial\Omega \end{cases}$$

- Least square minimization

$$J(A, B, n_e) = J_0 + K_1 J_1 + K_2 J_2 + J_\epsilon$$

with

$$J_0 = \sum_j (\frac{1}{r}\frac{\partial\psi}{\partial n}(N_j) - h_j)^2$$

$$J_1 = \sum_i (\int_{C_i} \frac{n_e}{r}\frac{\partial\psi}{\partial n} dl - \alpha_i)^2$$

$$J_2 = \sum_i (\int_{C_i} n_e dl - \beta_i)^2$$

$$J_\epsilon = \epsilon \int_0^1 (\frac{\partial^2 A}{\partial\bar{\psi}^2})^2 d\bar{\psi} + \epsilon \int_0^1 (\frac{\partial^2 B}{\partial\bar{\psi}^2})^2 d\bar{\psi} + \epsilon_{ne} \int_0^1 (\frac{\partial^2 n_e}{\partial\bar{\psi}^2})^2 d\bar{\psi}$$

Numerical method

Finite element resolution

$$\begin{cases} \text{Find } \psi \in H^1 \text{ with } \psi = g \text{ on } \partial\Omega \text{ such that} \\ \forall v \in H^1_0, \int_\Omega \frac{1}{\mu_0 r} \nabla\psi\nabla v dx = \int_{\Omega_p} \lambda[\frac{r}{R_0} A(\bar{\psi}) + \frac{R_0}{r} B(\bar{\psi})] v dx \end{cases}$$

with

$$A(x) = \sum_i a_i f_i(x), \quad B(\psi) = \sum_i b_i f_i(x), \; u = (a_i, b_i)$$

Fixed point

$$K\psi = Y(\psi)u + g$$

K modified stiffness matrix, u coefficients of A and B, g Dirichlet BC

Direct solver : $(\psi^n, u) \rightarrow \psi^{n+1}$

$$\psi^{n+1} = K^{-1}[Y(\psi^n)u + g]$$

Numerical method

Least-square minimization

$$J(u) = \|C(\psi)\psi - d\|^2 + u^T A u$$

- d : experimental measurements
- A : regularization terms

Approximation

$J(u) = \|C(\psi^n)\psi - d\|^2 + u^T A u$, with $\psi = K^{-1}[Y(\psi^n)u + g]$

$$\begin{aligned} J(u) &= \|C(\psi^n)K^{-1}Y(\psi^n)u + C(\psi^n)K^{-1}g - d\|^2 + u^T A u \\ &= \|E^n u - F^n\|^2 + u^T A u \end{aligned}$$

Normal equation. Inverse solver : $\psi^n \rightarrow u$

$$(E^{nT}E^n + A)u = E^{nT}F^n$$

Algorithm. EQUINOX

A pulse in real-time :

- Quasi-static approach :
  - first guess at time t = equilibrium at time $t - \delta t$
  - limited number of fixed-point iterations
- Normal equation : $\approx$ 10 Bspline basis func.
  $\rightarrow$ small $\approx 20 \times 20$ linear system
- Tikhonov regularization parameters unchanged
- $K = LU$ and K^{-1} precomputed and stored once for all
- Expensive operations : update products $C(\psi)K^{-1}$ and $C(\psi)K^{-1}Y(\psi)$

J. Blum, C. Boulbe, B. Faugeras,*Reconstruction of the equilibrium of the plasma in a Tokamak and identification of the current density profile in real time*, Journal of Computational Physics, Elsevier, 2012, 231, pp.960-980

Algorithm verification : twin experiments

Method

- Functions A and B given. Generate "measurements" with direct code
- Test the possibility to recover the functions by solving the inverse problem

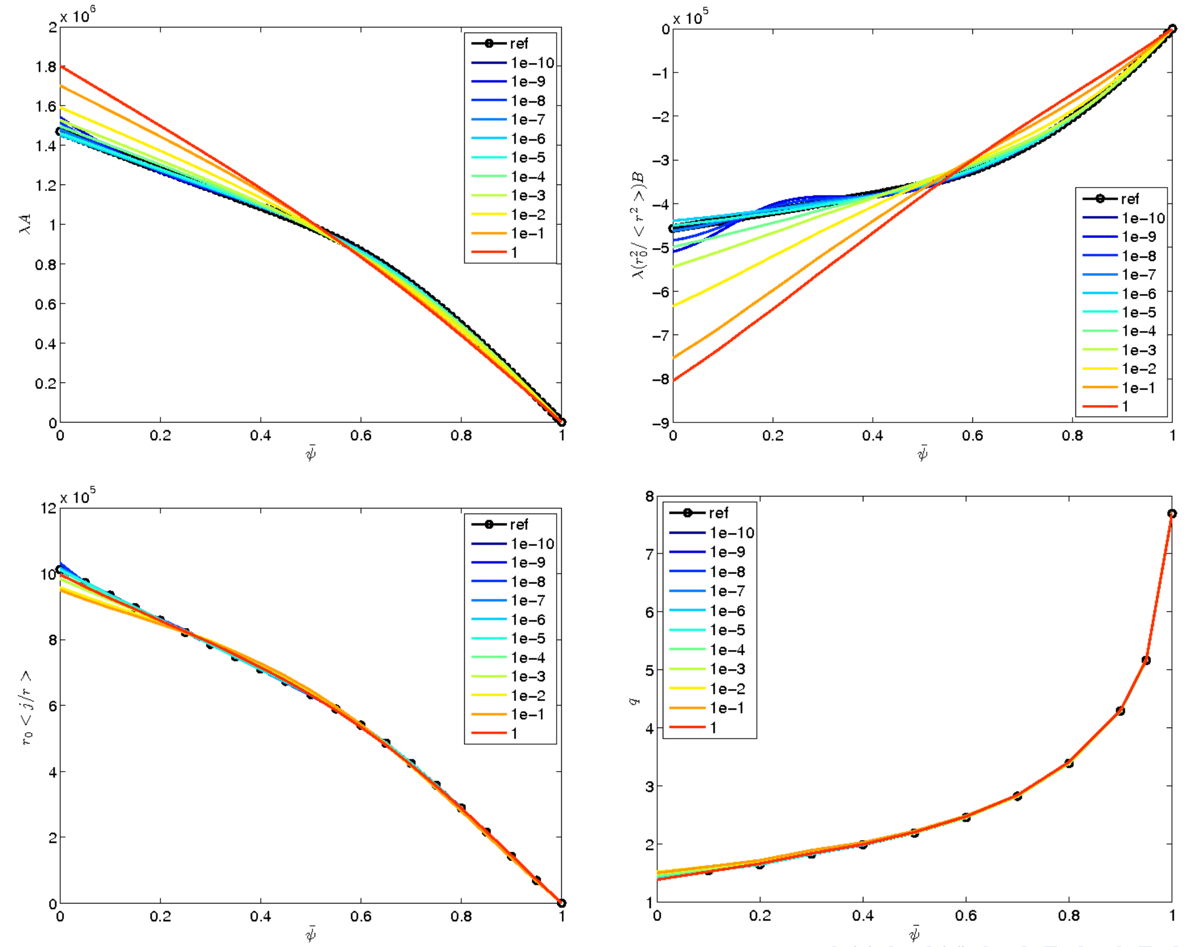
Noise free experiments. Magnetics only.

- With a well-chosen regularization parameter ε , A and B are well recovered.
- Averaged current density and q profiles are not very sensitive to ε.

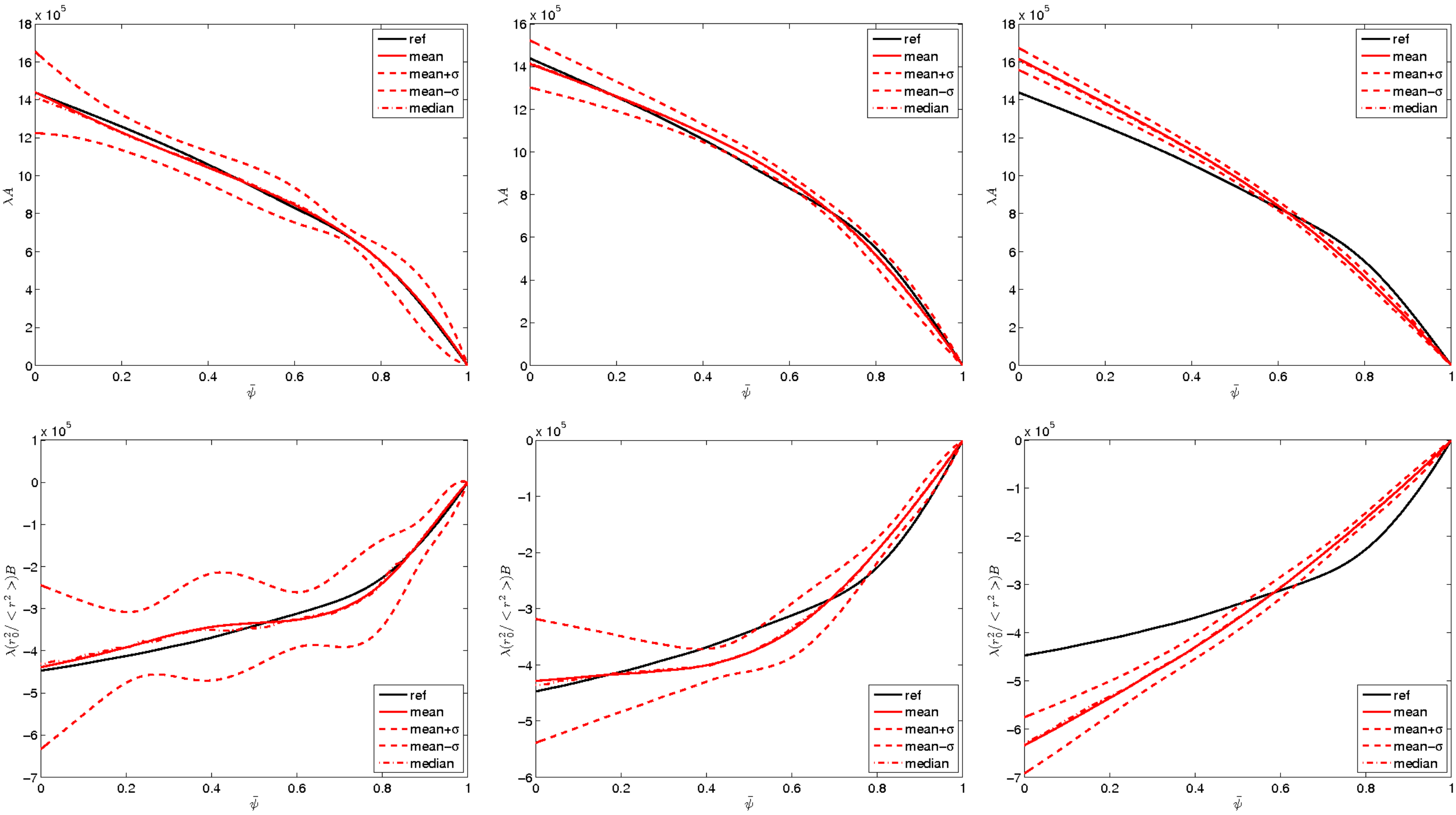
Experiments with noise. Magnetics only and mag+polarimetry.

- Averaged current density and q profiles are less sensitive to noise than A and B.
- With polarimetry A and B are better constrained.

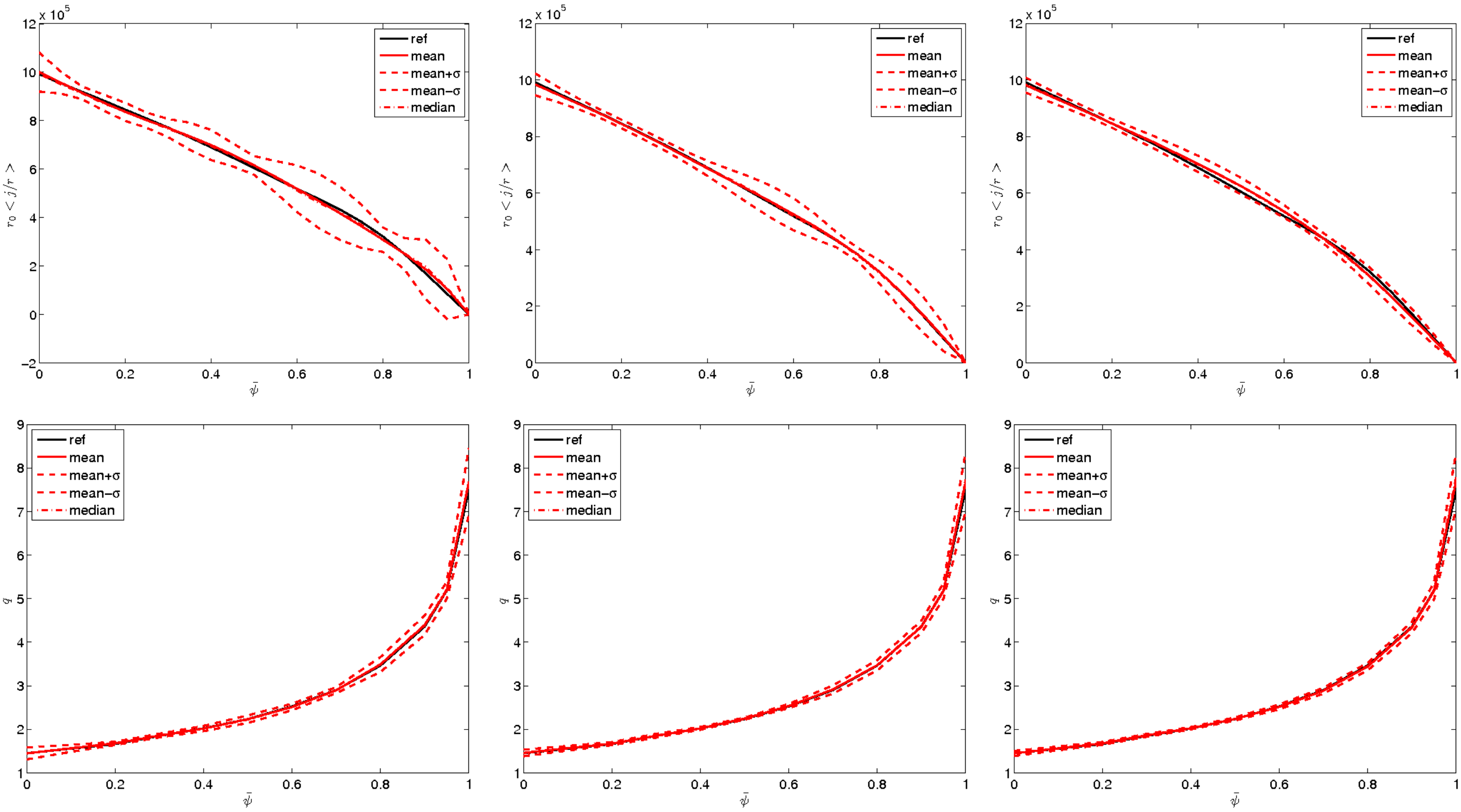
Noise free twin experiment. Magnetics only. Identified A, B, $r_0 < \dfrac{j(r,\bar{\psi})}{r} >$ and q for different ε



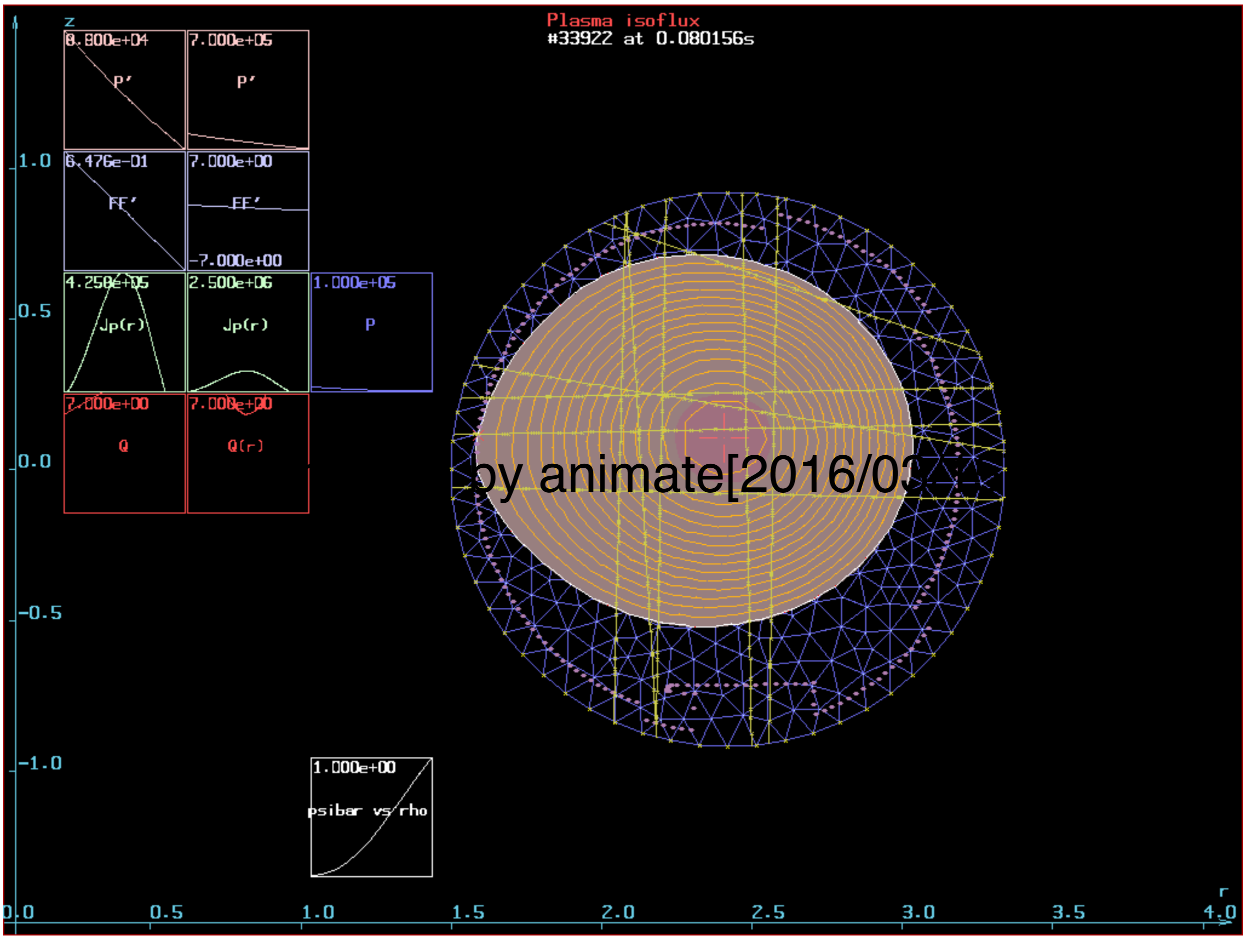
1% noise twin exp. Mag. and polar. Mean $\pm$ stand. dev. (200 exp.) identified A and B for $\varepsilon = 0.01,\ 0.1,\ 1$



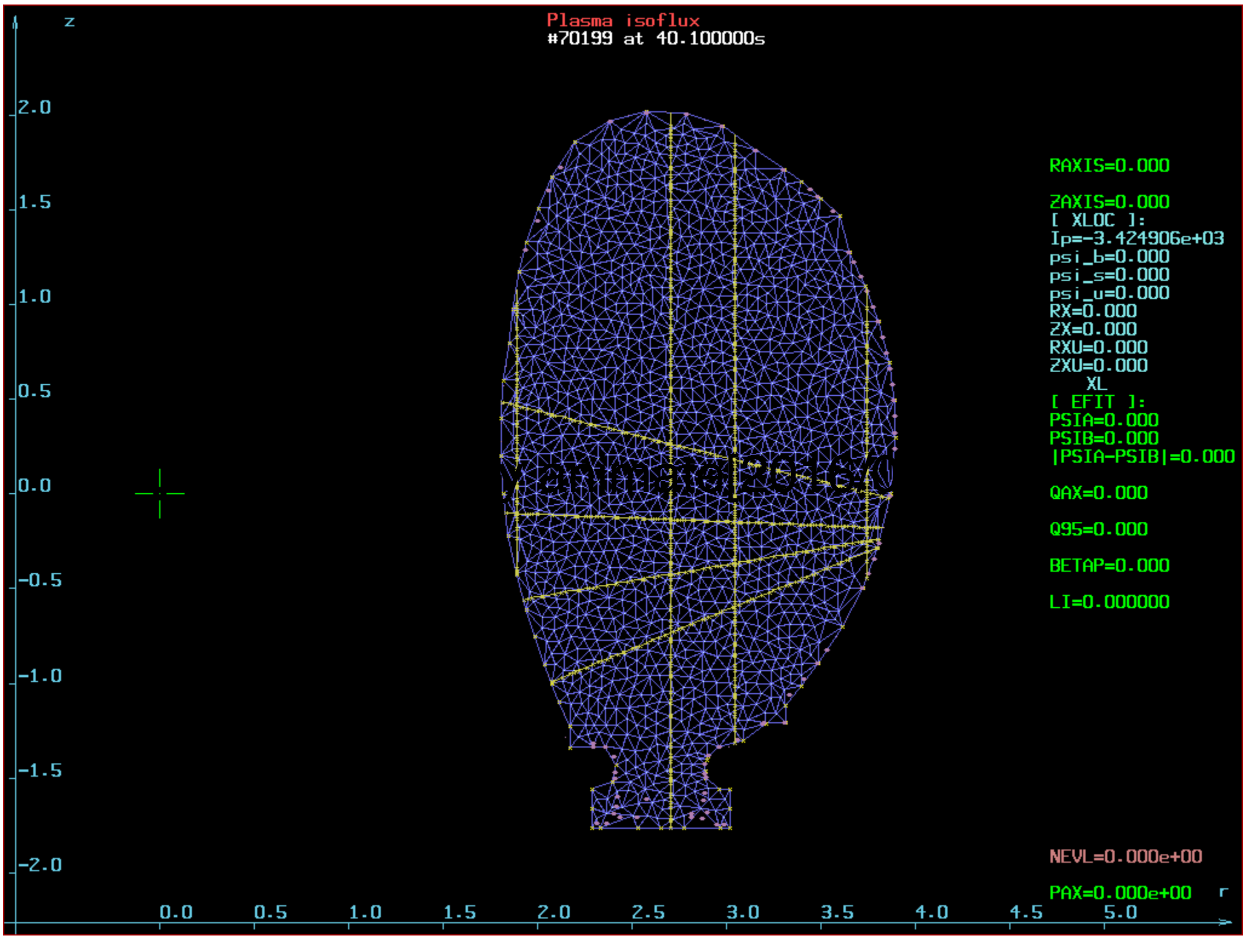
Same for mean current density and safety factor



Tore Supra - Magnetics and polarimetry



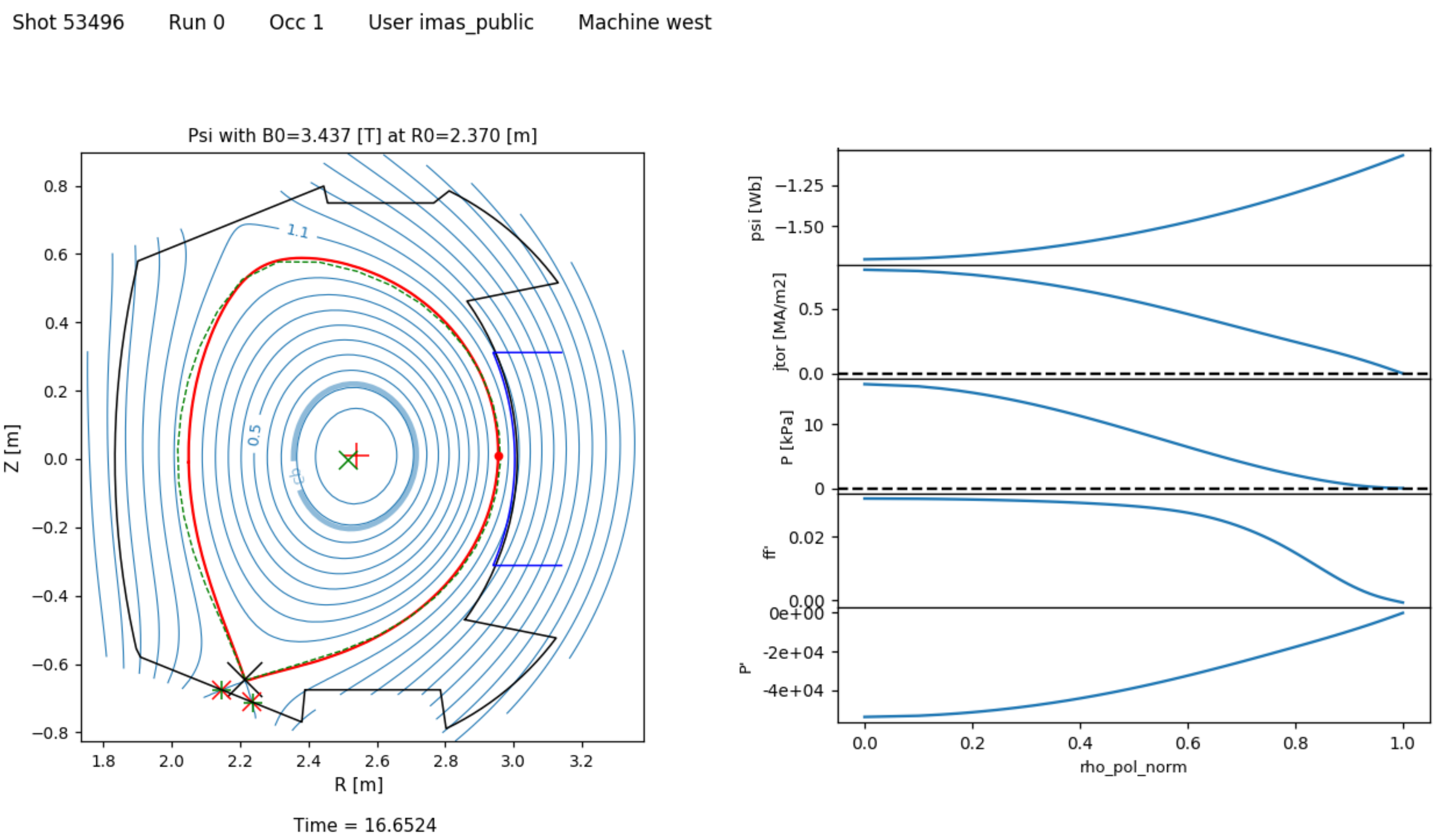
JET - Magnetics and polarimetry



NICE : N-ewton direct and I-nverse C-omputation for E-quilibrium

- merge in a single performant modern C++ code the numerical methods from
  - VacTH (toroidal harmonics in vacuum)
  - Equinox (equilibrium reconstruction in bounded domain)
  - Cedres++(equilibrium computation in full domain)
- add new numerical methods among which variants of SQP for optimization, and the possibility to use polarimetry with the Stokes model for equilibrium reconstruction
- final aim is to have a unified, complete and modular code for direct and inverse equilibrium computations
- Status :
  - mature for equilibrium reconstruction, direct and inverse static equilibrium, direct evolution
  - used at WEST for equilibirium recontruction
  - tested on TCV, AUG, JET and ITER
  - IMAS compatible

NICE used at WEST for equilibrium reconstruction



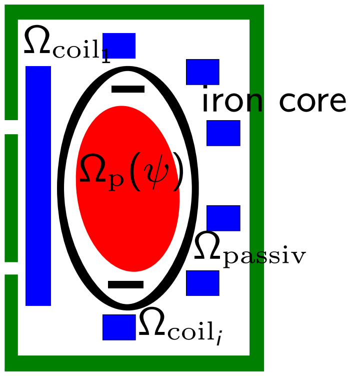
Optimization of plasma scenarios

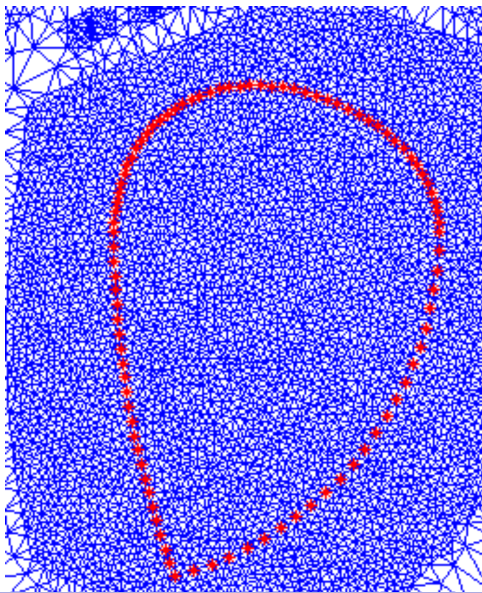
The inverse stationary problem :

Objective ($N_{\text{desi}} + 1$ points (r_i, z_i) given) and regularization :

$$K(\psi) := \frac{1}{2} \sum_{i=1}^{N_{\text{desi}}} \big(\psi(r_i, z_i) - \psi(r_0, z_0)\big)^2$$

$$R(I_1, \dots, I_L) := \sum_{i=1}^{L} \frac{w_i}{2} I_i^2$$





Optimization of plasma scenarios

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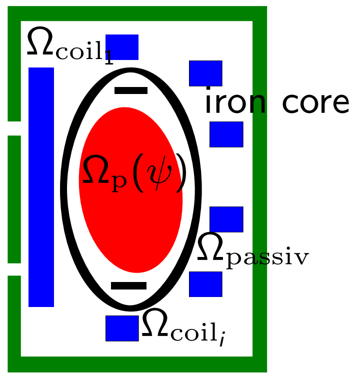


$$K(\psi) := \frac{1}{2} \sum_{i=1}^{N_{\mathrm{desi}}} \big(\psi(r_i, z_i) - \psi(r_0, z_0)\big)^2$$

$$R(I_1, \ldots, I_L) := \sum_{i=1}^{L} \frac{w_i}{2} I_i^2$$

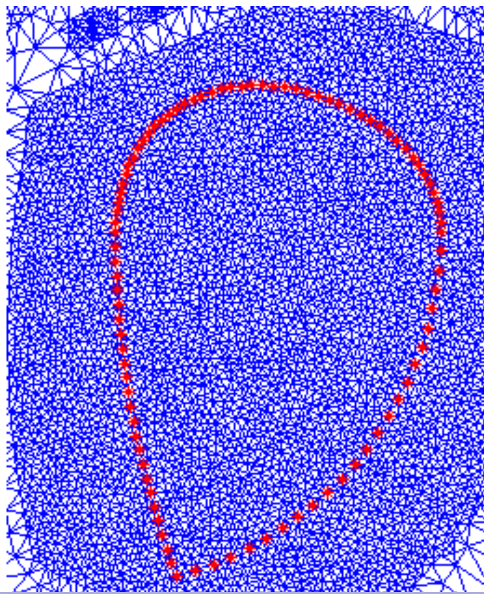
Optimal Control/Inverse Problem :

$$\min_{\psi, I_1, \ldots I_L} K(\psi) + R(I_1, \ldots I_L)$$

subject to

$$-\nabla \cdot \left(\frac{1}{\mu r} \nabla \psi\right) = \begin{cases} r S_{p'}(\psi_{\mathrm{N}}) + \frac{1}{\mu_0 r} S_{ff'}(\psi_{\mathrm{N}}) & \text{in } \Omega_{\mathrm{p}}(\psi)\,, \\ I_i / |\Omega_{\mathrm{coil}_i}| & \text{in } \Omega_{\mathrm{coil}_i}\,, \\ 0 & \text{elsewhere}\,, \end{cases}$$

$$\psi(0, z) = 0\,, \qquad \lim_{\|(r,z)\| \to +\infty} \psi(r, z) = 0\,,$$



The evolution problem : optimal voltage $\vec{V}(t)$

Objective (evolution of $N_{\text{desi}} + 1$ points (r_i, z_i) given) and regularization :

$$K(\psi(t)) := \frac{1}{2}\int_0^T \left(\sum_{i=1}^{N_{\text{desi}}} \left(\psi(r_i(t), z_i(t), t) - \psi(r_0(t), z_0(t), t)\right)^2\right) dt\,,$$

$$R(\vec{V}(t)) := \sum_{i=1}^{L} \frac{w_i}{2}\int_0^T V_i^2(t)dt\,.$$

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$$R(\vec{V}(t)) := \sum_{i=1}^{L} \frac{w_i}{2}\int_0^T V_i^2(t)dt\,.$$

Optimal Control/Inverse Problem :

$$\min_{\psi(t),\vec{V}(t)} K(\psi(t)) + R(\vec{V}(t))$$

subject to

$$-\nabla\cdot\left(\frac{1}{\mu r}\nabla\psi\right) = \begin{cases} rS_{p'}(\psi_{\mathrm{N}},t) + \frac{1}{\mu_0 r}S_{ff'}(\psi_{\mathrm{N}},t) & \text{in } \Omega_{\mathrm{p}}(\psi)\,, \\ |\Omega_{\mathrm{coil}_i}^{-1}|\left(\boldsymbol{S}\vec{V}(t) + \boldsymbol{R}\vec{\Psi}(\partial_t\psi)\right)_i & \text{in } \Omega_{\mathrm{coil}_i}\,, \\ -\frac{\sigma_k}{r}\partial_t\psi & \text{in } \Omega_{\mathrm{passive}}\,, \\ 0 & \text{elsewhere}\,, \end{cases}$$

$$\psi(0,z,t) = 0\,,\quad \lim_{\|(r,z)\|\to+\infty}\psi(r,z,t) = 0\,,\quad \psi(r,z,0) = \psi_0(r,z)\,,$$

PDE-constrained optimization with non-linear constraints.

Weak Formulation, Stationary Problem

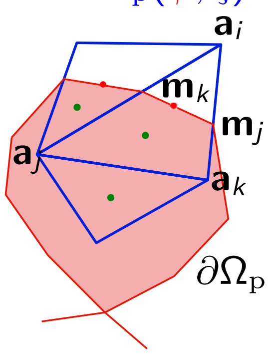
Find $\psi \in V$ such that

$$\mathsf{A}(\psi,\xi) - \mathsf{J}_{\mathrm{p}}(\psi,\xi) + \mathsf{c}(\psi,\xi) = \ell(\vec{I},\xi) \quad \forall \xi \in V\,.$$

with
$$V = \left\{ \psi : \Omega \to \mathbb{R}, \int_\Omega \psi^2 r^{-1}\, drdz < \infty, \int_\Omega |\nabla\psi|^2 r^{-1}\, drdz < \infty \right\} \;,$$

$$\mathsf{A}(\psi,\xi) := \int_\Omega \frac{1}{\mu r} \nabla\psi \cdot \nabla\xi \, drdz\,, \qquad \ell(\vec{I},\xi) := \sum_{i=1}^{N_{\mathrm{coil}}} |\Omega_{\mathrm{coil}_i}|^{-1} \vec{I}_i \int_{\Omega_{\mathrm{coil}_i}} \xi \, drdz\,,$$

$$\mathsf{J}_{\mathrm{p}}(\psi,\xi) := \int_{\Omega_{\mathrm{p}}(\psi)} \left(r S_{p'}(\psi_{\mathrm{N}}(\psi)) + \frac{1}{\mu_0 r} S_{ff'}(\psi_{\mathrm{N}}(\psi)) \right) \xi \, drdz\,,$$



$$\mathsf{c}(\psi,\xi) \approx \int_{\partial\Omega} \xi\, \partial_{\mathbf{n}}\psi \, dS \text{ for boundary condition at infinity}\,.$$

The domain Ω is semi-circle with radius ρ.

Sequential quadratic programming method

Minimization problem

$$\min_{\Psi,u} \frac{1}{2}\Psi^T K \Psi + \frac{1}{2}u^T H u \quad \text{s.t.} \quad B(\Psi) = F(u)$$

with $\Psi = (\psi_1, ..., \psi_{n_nodes})^T$ et $u = (u_1, ..., u_N)^T$.
u represents the PF currents I_i in the stationary problem and the voltages V_i in the evolutive problem.

Lagrangian

$$L(\Psi, u, p) = \frac{1}{2}\Psi^T K \Psi + \frac{1}{2}u^T H u + p^T(B(\Psi) - F(u))$$

Stationary point of the Lagrangian

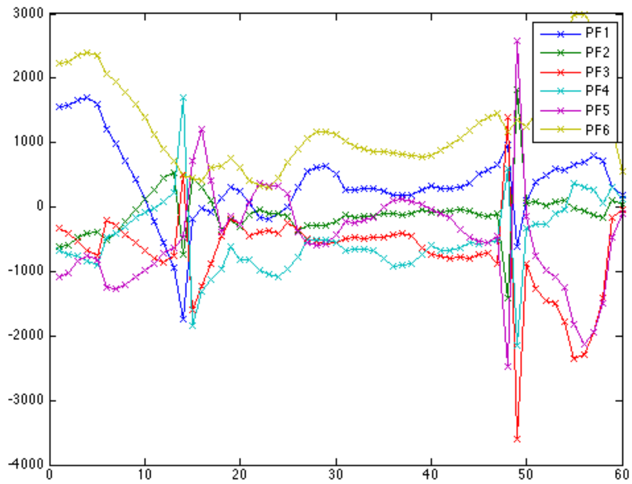
$$\begin{aligned} K\Psi + D_\Psi B^T(\Psi)p &= 0, \\ Hu - D_u F^T(u)p &= 0, \\ B(\Psi) - F(u) &= 0. \end{aligned}$$

SQP is equivalent to the resolution of this system by Newton's iterations.

H. Heumann & al., *Quasi-static Free-Boundary Equilibrium of Toroidal Plasma with CEDRES++ : Computational Methods and Applications*, Journal of Plasma Physics, Cambridge University Press (CUP), 2015, pp.35.

Control of Transient Plasma Equilibrium, ITER

Voltages at 60 timesteps :

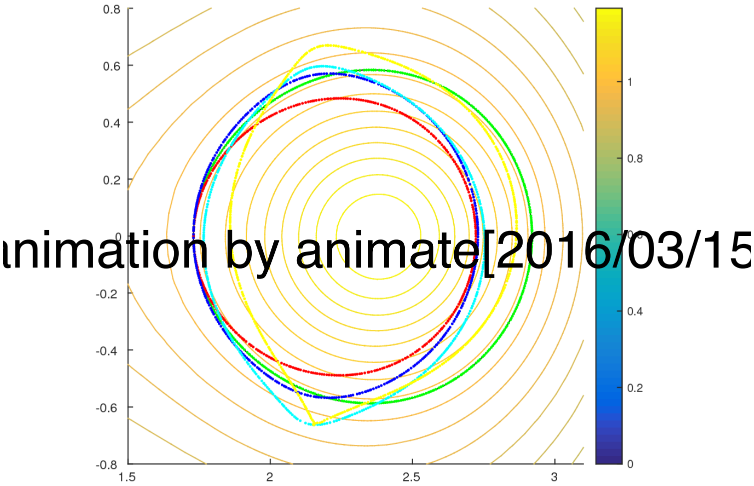


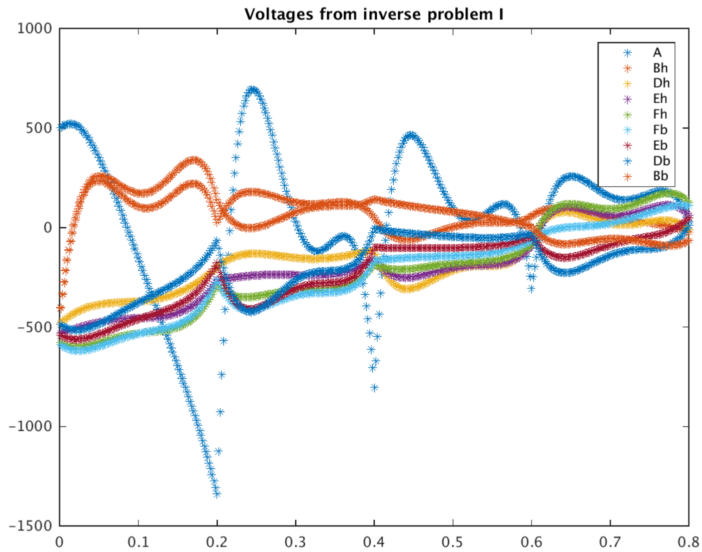
Control of Transient Plasma Equilibrium, WEST

Objective function (desired shape at final time T) :

$$K(\psi(t)) := \frac{1}{2}\left(\sum_{i=1}^{N_{\mathrm{desi}}} \left(\psi(r_i(T), z_i(T), T) - \psi(r_0(T), z_0(T), T)\right)^2\right),$$

Go from green to yellow desired boundary in passing red, blue and cyan !



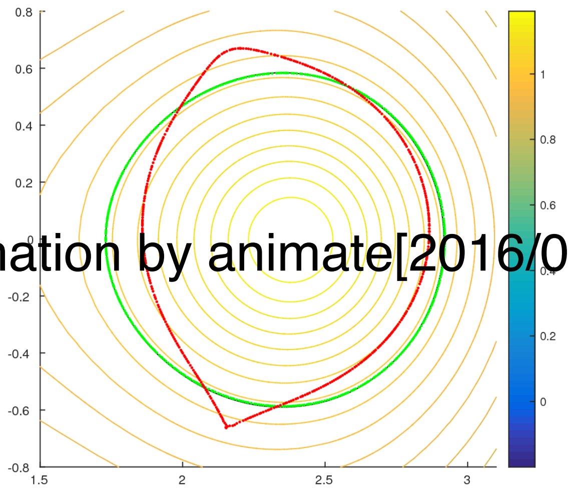


Control of Transient Plasma Equilibrium, WEST

Objective (desired shape at final time T) :

$$K(\psi(t)) := \frac{1}{2}\left(\sum_{i=1}^{N_{\mathrm{desi}}}\left(\psi(r_i(T), z_i(T), T) - \psi(r_0(T), z_0(T), T)\right)^2\right),$$

Go directly from green to red desired boundary !



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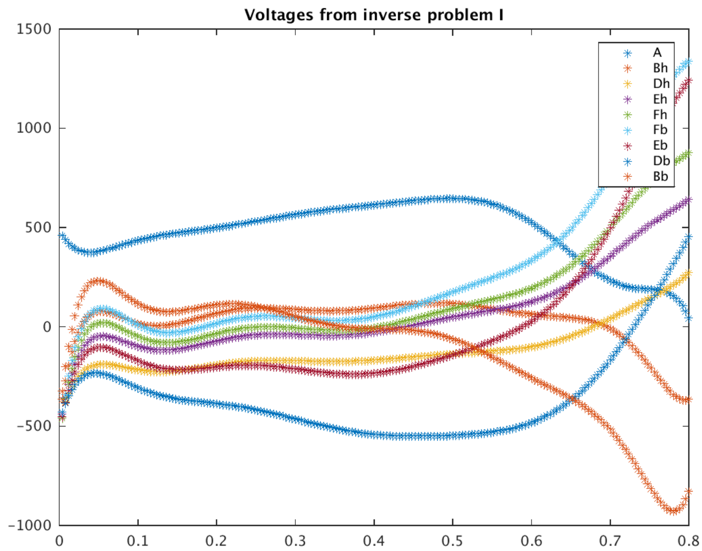

Perspective : MHD at the Resistive Timescale

Coupled problem   equilibrium (EQ) $\leftrightarrow$ resistive diffusion/transport (RD)

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Grad/Hogan '70 : *Queer differential equations.*

Perspective : MHD at the Resistive Timescale

Coupled problem equilibrium (EQ) $\leftrightarrow$ resistive diffusion/transport (RD)

Grad/Hogan '70 : *Queer differential equations.*

Find $\mathbf{y}_1, \mathbf{y}_2$ s.t.

$$EQ(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1) = 0 \quad \text{2D problem}$$
$$RD(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_2) = 0 \quad \text{1D problem}$$

Perspective : MHD at the Resistive Timescale

Coupled problem equilibrium (EQ) $\leftrightarrow$ resistive diffusion/transport (RD)

Grad/Hogan '70 : *Queer differential equations.*

Find $\mathbf{y}_1, \mathbf{y}_2$ s.t.

$$EQ(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1) = 0 \quad \text{2D problem}$$
$$RD(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_2) = 0 \quad \text{1D problem}$$

with

- state variables *flux* $\psi \approx \mathbf{y}_1$ and *current* $\mathbf{j}_{\text{plasma}} \approx \mathbf{y}_2$;
- control variables in EQ : voltages $\approx \mathbf{u}_1$;
- control variables in RD : beam injection, wave heating, ... $\approx \mathbf{u}_2$;

Perspective : MHD at the Resistive Timescale

Coupled problem   equilibrium (EQ) $\leftrightarrow$ resistive diffusion/transport (RD)

Grad/Hogan '70 :   *Queer differential equations.*

Find $\mathbf{y}_1, \mathbf{y}_2$ s.t.

$$EQ(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1) = 0 \qquad \text{2D problem}$$
$$RD(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_2) = 0 \qquad \text{1D problem}$$

with

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Optimal control for coupled problem

$$\min_{\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1, \mathbf{u}_2} C(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1, \mathbf{u}_2) \qquad \textit{deviation from desired state}$$

s.t.
$$EQ(\mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1) = 0 \qquad \textit{equilibrium}$$
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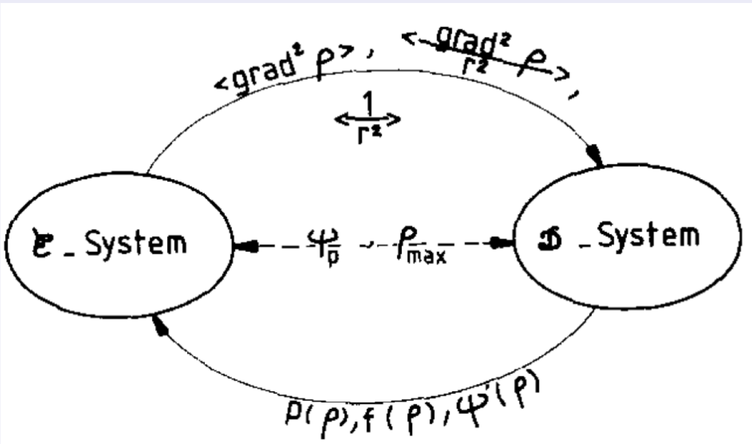
$\Longrightarrow$ We need reliable solver for (EQ,RD), derivatives, sensitivities !

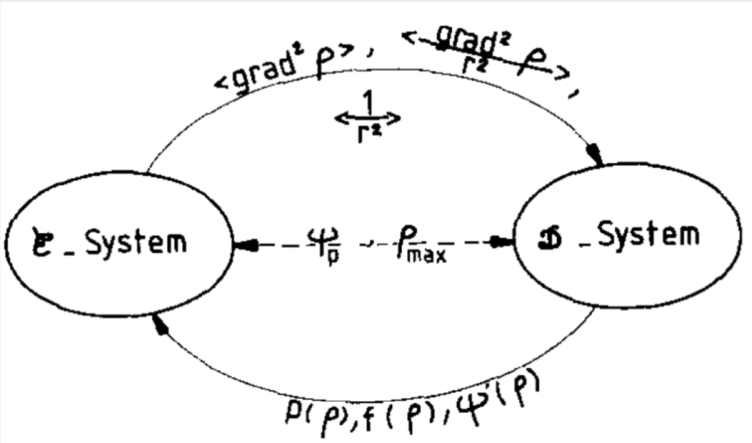
Resistive diffusion equation

Coupling



Resistive diffusion equation

Coupling

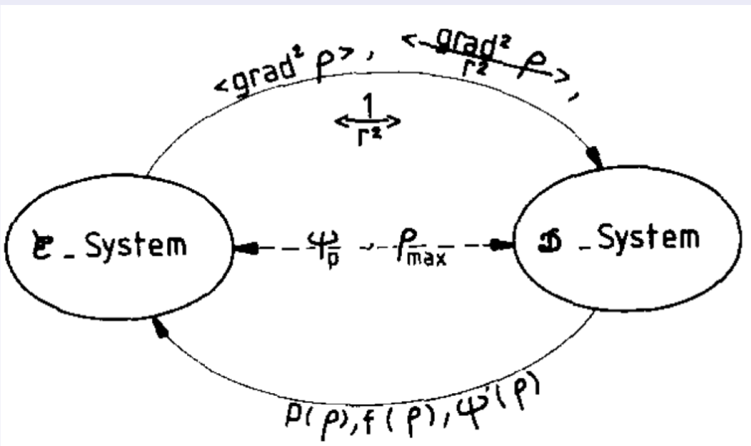


Resistive diffusion equation

RD via averaging over a certain label $\rho(s) = \sqrt{\int_{\psi > s} f(\psi)\, r\, dr\, dz}$:

$$\partial_t \overline{\psi}'(y,t) - \frac{1}{\mu_0} \frac{\partial}{\partial y} \left(\frac{\eta y}{C^2_{\rho,3}(y,t)} \frac{\partial}{\partial y} \left(\frac{C_{\rho,2}(y,t) C_{\rho,3}(y,t)}{y} \overline{\psi}'(y,t) \right) \right) = 0,$$

Coupling



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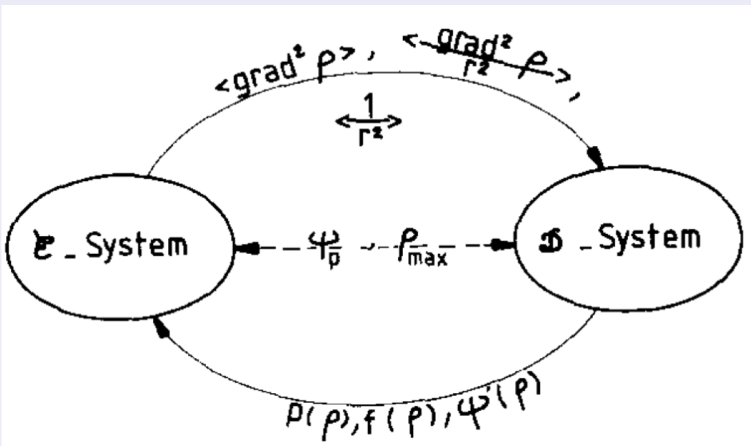
with geometric coefficients $\{h\}_\rho(y) := \int_{\{\rho(r,z)=y\}} \frac{h\, r\, dS}{|\nabla \rho|}$:

$$C_{\rho,3}(y,t) := \{1\}_\rho(y,t), \quad C_{\rho,3}(y,t) \quad := \{\frac{1}{r^2}\}_\rho(y,t), \quad C_{\rho,2}(y,t) := \{\frac{|\nabla \rho|^2}{r^2}\}_\rho(y,t).$$

Coupling



Conclusion

- The use of optimal control theory for systems governed by partial differential equations has enabled the resolution of inverse problems and the optimization of scenarios
- It has been used for the computation of feedforward control for the poloidal field system
- Control of ill-posed problems is well-posed (stable simulation of scenarios for elongated plasmas)
- Perspective : optimization for coupling equilibrium+ transport